Drag and diffusion co-efficients of heavy quarks in hard thermal loop approximations

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The drag and diffusion coefficients of heavy quarks propagating through quark gluon plasma (QGP) have been evaluated within the ambit of thermal field theory in Hard Thermal Loop (HTL) approximations. It is observed that the magnitudes of both the transport coefficients are changed significantly from their values obtained when the t channel divergence is shielded simply by Debye mass. The implications of this changes in the transport coefficients on the heavy ion phenomenology have been discussed.

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I. INTRODUCTION

The study of the transport coefficients of strongly coupled system is a field of high contemporary interest both theoretically and experimentally. In one hand the calculation of the lower bound on the shear viscosity (η) to entropy density (s) ratio (η/s) within the frame work of AdS/CFT model [1] has ignited enormous interests among the theorists. On the other hand, the experimental study of the η/s for cold atomic systems and QGP and their similarities have generated huge interest across various branches of physics (see [2] for a review). In general, the interaction of probes with a medium bring out useful information about the nature of the medium. Since the magnitude of the transport co-efficients are sensitive to the coupling strength hence these quantities can be adapted as useful quantities to characterize a medium. In the context of probing QGP expected to be produced in ultra-relativistic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies, we choose the heavy quarks (HQs), charm and beauty as probes. That is, we would like to extract the drag and diffusion coefficients of the QGP by studying the propagation of HQs through QGP. Selection of HQs as probes has several advantages, such as (i) they are produced very early in the collisions and remain extant throughout the evolution of the QGP. As a result, the HQs witness the entire evolution of the system. It is expected that the HQ thermalization time is larger than the light quarks by a factor m/T where m is the mass of the HQ and T is the temperature. Therefore, the HQs may remain out of equilibrium in QGP. (ii) That is the chances of HQs getting thermalized in the system is weaker and hence do not dictate the bulk properties of QGP. Moreover, the observed transverse momentum suppression (R_{AA}) of leptons originating from the decays of heavy flavours produced in nuclear collisions as compared to those produced in proton+proton (pp) collisions at the same colliding energy [3–5] offer us an opportunity to estimate the drag and diffusion coefficients of QGP. In the present circumstances we required to describe the motion of the non-equilibrated HQs in the background of equilibrated system of QGP. An appropriate foundation is provided by the Fokker-Planck (FP) equation [6, 7], which reads as follows:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(p)f + \frac{\partial}{\partial p_i}[B_{ij}f]\right] \tag{1}$$

where f stands for the momentum-space distribution of the particle (HQs here) undergoing Brownian motion in the thermal bath of QGP. The question of HQ thermalization can be addressed by comparing the solution of FP equation with the HQs thermal distribution at any given time. A_i and B_{ij} are related to the drag and diffusion co-efficients respectively. The interactions of the HQs with the QGP are incorporated in A_i and B_{ij}. That is A_i and B_{ij} can supply the information about the nature of the QGP [8–19]. The issue of HQ thermalization in QGP can also be addressed experimentally by measuring the elliptic flow (v_2) of leptons from the decays of HQs. Therefore, the evaluation of the drag and diffusion coefficients of QGP become extremely important. In the present work we attempt to estimate both these coefficients by using the techniques of thermal field theory in hard thermal loop (HTL) approximations.

We will see below that that drag (diffusion) co-efficients are, essentially, momentum (square of the momentum) transfer weighted over the squared interaction matrix element ($|M|^2$). This indicates that an accurate evaluation of $|M|^2$ is of vital importance. Present work contemplates on determining elastic transport co-efficients of QGP being probed by heavy quarks (charm and bottom) within the ambit perturbative QCD (pQCD) and HTL approximations.

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The two main elastic process which contribute to the transport coefficients are: $Q+q \rightarrow Q+q$ and $Q+g \rightarrow Q+g$. Here Q (q) stands for heavy (light) quarks and g denotes gluon. The $|M|^2$ for these processes contain t-channel divergence which are normally regulated by introducing thermal mass (m_D) for the exchanged gluons [12, 17] i.e. by replacing t by $t - m_D^2$ in the denominator of the matrix elements. In the present work, instead of shielding the divergences simply by (static) Debye mass we will use the HTL approximated gluon propagator in the t-channel diagrams in a self-consistent way.

The paper is organized as follows. In the next section the general expressions for the drag and diffusion coefficients are outlined. In section III we briefly discuss the main steps for evaluating the gluon propagators in HTL approximations. Section IV is devoted for presenting results on the drag and diffusion coefficients. The summary and conclusions of the present work is presented in section V. The appendix contains the detailed derivation of the matrix elements required for the evaluation of drag and diffusion coefficients without small angle approximation.

II. THE DRAG AND DIFFUSION CO-EFFICIENTS

In terms of the transition rates the collision integral of the Boltzmann transport equation can written as [6]:

$$\left[\frac{\partial f}{\partial t}\right]_{collisions} = \int d^3\mathbf{k}[w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k}) - w(\mathbf{p},\mathbf{k})f(\mathbf{p})]. \tag{2}$$

where $w(\mathbf{p},\mathbf{k})$ is collision rate, say for the processes, $\mathbf{Q}(\mathbf{p}) + \mathbf{g}(\mathbf{q}) \rightarrow \mathbf{Q}(\mathbf{p}-\mathbf{k}) + \mathbf{g}(\mathbf{q}+\mathbf{k})$. Using Landau approximation *i.e.* by expanding $w(\mathbf{p}+\mathbf{k},\mathbf{k})$ in powers of $\mathbf{k}$ and keeping upto quadratic term, the Boltzmann transport equation can be written as [12, 17]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})f]\right] \quad , \tag{3}$$

where the kernels are defined as

$$A_i = \int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})k_i \quad , \tag{4}$$

and

$$B_{ij} = \frac{1}{2}\int d^3\mathbf{k} w(\mathbf{p},\mathbf{k})k_i k_j. \tag{5}$$

For $|\ \mathbf{p}\ | \rightarrow \mathbf{0}$, $A_i \rightarrow \gamma p_i$ and $B_{ij} \rightarrow D\delta_{ij}$ where γ and D stand for drag and diffusion co-efficients respectively. The drag and diffusion coefficients have recently been evaluated within the ambit of AdS/CFT [20] and pQCD [21] and their importance for jet quenching have been discussed. Eq. 3 is a nonlinear integro-differential equation known as the Landau kinetic equation. The appearance of parton distribution in the expression for ω makes Eq. 3 a non-linear one. For the problem under consideration one of the colliding partners (light quarks or gluons) is in equilibrium. In such a situation the distribution function which appears in w can be replaced by thermal distribution. As a consequence Eq.3 becomes a linear partial differential equation, known as Fokker-Planck (FP) equation. The T dependence of the transport coefficients enter through the thermal distribution appearing in ω.

As mentioned above the drag and diffusion coefficients are related to the expressions for A_i and B_{ij}. These coefficients can be calculated from the following expression [12] with appropriate choice of the function $F(p')$,

$$\begin{aligned}\langle\langle F(p)\rangle\rangle &= \frac{1}{512\pi^4}\frac{1}{E_p}\int_0^\infty qdqd(cos\chi)\frac{s-m^2}{s}f(q)\int_{-1}^1 \\ &\quad d(cos\theta_{c.m.})\frac{1}{g_Q}\overline{|M|}^2\int_0^{2\pi} d(\phi_{c.m.})F(p')\end{aligned} \tag{6}$$

where g_Q is the HQ degeneracy, $F(p' = p - k)$ is a function of $p,\ q$ and CM frame scattering angles and $cos\chi$ can be obtained from,

$$s = p^2 + q^2 + 2(E_pE_q - |\vec{p}||\vec{q}|cos\chi) \tag{7}$$

$\theta_{c.m.}$ and $\phi_{c.m.}$ are polar and azimuthal angles of q respectively. Drag (γ) can be obtained by the following replacement in Eq. 6:

$$F(p') = 1 - \frac{p.p'}{p^2} \tag{8}$$

For determining diffusion (D) we substitute,

$$F(p') = \frac{1}{4}\left[p'^2 - \frac{(p.p')^2}{p^2}\right] \tag{9}$$

III. HTL APPROXIMATION AND RESUMMED GLUON PROPAGATOR

As discussed before the calculation of drag and diffusion co-efficients involve the evaluation of amplitudes for processes like $Q + q \to Q + q$ and $Q + g \to Q + g$ [22]. The amplitudes from bare perturbation theory contains t-channel divergence due to low four-momentum, $Q = (\omega, \vec{q})$ gluon exchange. This divergence can be regulated by introducing thermal mass of gluon. Here, we study the HTL approximations [23] and re-summation of gluon propagator which will enable us to regulate the t-channel divergence in a self-consistent way and hence will lead to comparatively more reliable values of the transport co-efficients.

Our aim is to find out HTL approximated self-energy of gluon which goes as an input to the resummed gluon propagator to be used as effective thermal propagator regularizing the t channel divergence. The gluon self-energy in HTL approximation is discussed in detail in Ref. [24, 25]. In this section we give only an outline of the scheme. There are four diagrams which contribute to gluon self-energy (Fig. 1). The loop integrations can be written down

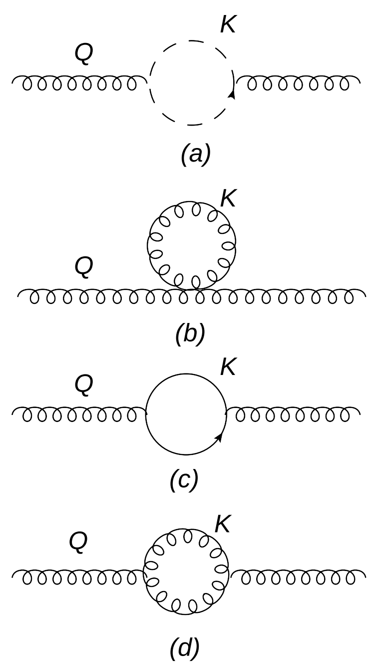

FIG. 1: Feynman diagrams contributing to gluon self-energy. (a)ghost-gluon loop. (b)four-gluon vertex. (c) quark-antiquark pair creation. (d)three-gluon vertex.

easily if we keep in mind that the loop-momentum, $K = (k_0, \vec{k})$ is 'hard' compared to external gluon momentum, *i.e.* $K >> Q$ which enables us to use simplified three-gluon vertex [24]. Our goal will be to find out T^2 contributions of self-energy because the momentum integration is cut-off at the momentum scale$\sim T$ due to the presence of thermal distribution function. That is we can take-up the momentum integration $\int_0^\infty kdk$ which blows up as $k \to \infty$, but, insertion of distribution function makes it finite even at $k \to \infty$,

$$\int_0^\infty kf(k)dk = \frac{\pi^2 T^2}{12} \tag{10}$$

The leading contribution in Eq. 10 is given by $k \sim T$. If we are interested in high-temperature limit we can assume $|\vec{k}| >> |\vec{q}|$ and approximate related quantities accordingly [24].

The effective gluon propagators evaluated in one loop order in HTL approximation enter in the transport coefficients evaluated for the processes displayed Figs. 8 and 9.

For low momentum transfer (i.e.$\sim gT$ where g is the colour charge,$g = \sqrt{4\pi\alpha_s}$, α_s is the strong coupling), one has to use the resummed propagator [26]. The resummed gluon propagator is given by:

$$\Delta^{\mu\nu} = \frac{\mathcal{P}_T^{\mu\nu}}{-Q^2 + \Pi_T} + \frac{\mathcal{P}_L^{\mu\nu}}{-Q^2 + \Pi_L} + (\alpha - 1)\frac{Q^\mu Q^\nu}{Q^2} \tag{11}$$

will need HTL approximated Π_L and Π_T. The expressions for Π_L and Π_T in the context of effective propagator are discussed in the Appendix.

IV. RESULTS

The drag and diffusion coefficients due to elastic collision have been evaluated by using the HTL approximated gluon propagators discussed above, they are denoted by γ_{HTL} and D_{HTL} respectively. The corresponding quantities evaluated by shielding the divergence in the t-channel gluon propagator by Debye mass will be denoted by γ and D. We use zero-temperature Fermionic propagator for heavy quarks because they remain out of equilibrium in the system under consideration.

The drag co-efficients of charm and beauty quarks moving in QGP has been evaluated by applying HTL resummed gluon propagator. The variations of drags with temperature for HQs with momentum, $p = 1$ GeV as a function of temperature are displayed in Fig. 2. The results clearly indicate an enhancement and rapid variation of γ_{HTL} compared to γ. The increase is more prominent for charm than beauty. We have explicitly checked that in the static limit ($\omega/q = x \to 0$) the γ_{HTL} approaches γ. The variation of γ_{HTL} and γ with momentum is depicted in Fig. 3 for $T = 300$ MeV. The γ_{HTL} is greater than γ for the entire momentum range considered here. Again, drag being the measure of energy loss, increase in drag results in more suppression of heavy flavours measured at RHIC and LHC energies. From Fig. 2 we observe that at 400 MeV temperature the γ_{HTL} for charm quark is $\sim$33% more than the γ. Whereas, the corresponding difference is $\sim$25% for bottom quarks. We also observe that this difference increases with the increase in temperature. The momentum dependence of drag is distinctly affected if we consider the HTL resummation technique. For a 5 GeV charm the γ_{HTL} is $\sim$50% more than γ. For higher momenta (10 GeV) the difference reduces to $\sim$45%. In Fig. 4 the interaction rate of the HQs with the QGP is shown as function of temperature. The rate is increased when the HTL corrections to the t-channel gluon is taken into account. The inclusion of effective thermal gluon propagator increases the likelihood of charm or bottom being equilibrated with the medium. Therefore, this result will have important implications for elliptic flow of heavy flavours.

In Figs. 5 and 6 the diffusion coefficients D_{HTL} and D are plotted with T (for $p = 1$ GeV) and p (for $T = 300$ MeV) respectively. Similar to drag we observe that the difference between D_{HTL} and D at high momentum is significant. Diffusion co-efficients seem to be more sensitive to the use of effective propagator in a sense that we observe $\sim$100% change between the D_{HTL} and D at $T = 400$ MeV and this difference increases with T. Though unlike drag, this difference is not much ($\sim$3.5%) for a difference in charm and beauty quark masses. The momentum dependence of diffusion is significantly modified too. A 5 GeV charm diffuses $\sim$80% more when the exchanged gluon contains the HTL approximated changes in its spectral function. These changes in drag and diffusion co-efficients originate from the spectral modification of the t-channel gluons due to its interaction with the thermal bath. In the static limit $\pi_T \to 0$ and $\pi_L \to m_D^2$. The appearance of non-zero π_T makes γ_{HTL} larger than γ.

Now some comments on the magnitude of the values of D_{HTL} and D are in order here. The value of the diffusion coefficients in spatial co-ordinate, D_x^{HTL} can be estimated from the value of drag by using the relation $D_x^{HTL} = T(m\gamma_{HTL})^{-1}$. In Fig. 7 we plot D_x^{HTL} multiplied by the inverse of the thermal de Broglie length, $\Lambda = 1/(2\pi T)$. The results clearly indicate that the D_x remains well above the quantum bound.

V. SUMMARY AND CONCLUSION:

In summary, we have taken into account the thermal modifications of the gluon spectral function obtained within the ambit of HTL approximations in evaluating the drag and diffusion coefficients of HQs propagating through the QGP. The deviations between γ_{HTL} and γ and D_{HTL} and D is found to be substantial. The enhanced drag will result in higher suppression of the HQ momentum spectrum. The increase in drag will also enhance the chances of HQ getting equilibrated with the bulk of the system. These results will have crucial consequences on the observables like nuclear suppressions, $R_{AA}(p_T)$ and elliptic flow, v_2 of heavy flavours measured at RHIC and LHC experiments.

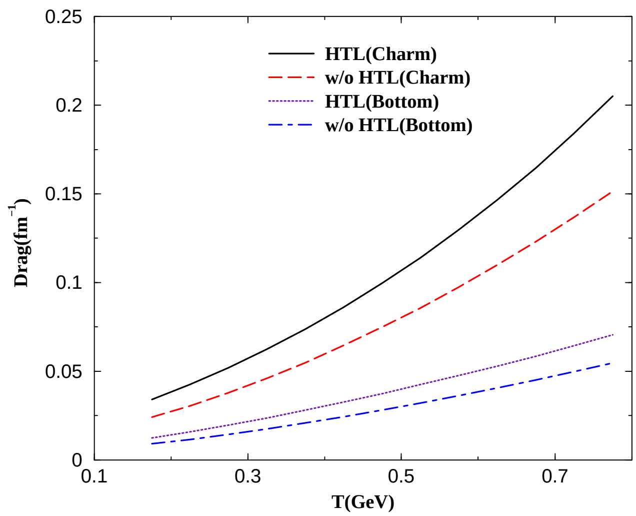

FIG. 2: (Color online) Variation of drag of heavy quarks of momentum 1 GeV with temperature.

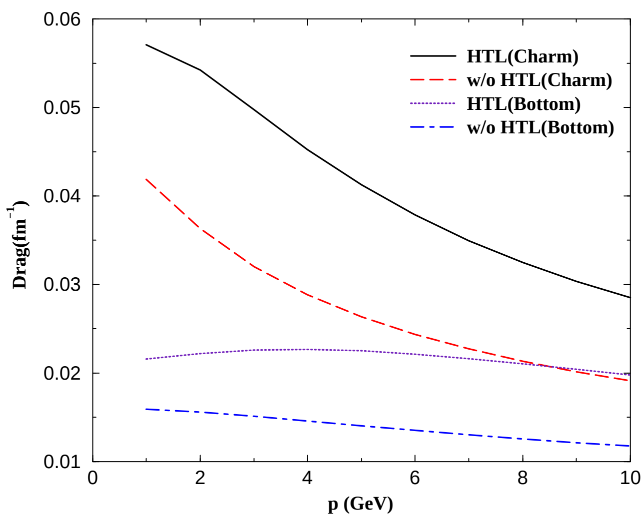

FIG. 3: (Color online) Variation of drag of heavy quarks with momentum in a QGP bath of temperature 300 MeV.

VI. APPENDIX:

In this appendix we demonstrate the calculation of matrix elements of the processes $Qq \to Qq$ (Fig. 8) by applying HTL approximation. The matrix elements for the process, $Qg \to Qg$ (Fig. 9) can be evaluated in a similar way. Before decomposing the gluon self energy into longitudinal and transverse components in the medium we define the following useful quantities [24]. Let u_μ be the fluid four-velocity (with normalization condition $u_\mu u^\mu = 1$) then any four-vector Q^μ, can be decomposed into a part which is parallel to fluid velocity and the other perpendicular to fluid velocity as:

$$\begin{aligned} \omega &= Q.u \\ \tilde{Q}_\mu &= Q_\mu - u_\mu(Q.u) \end{aligned} \tag{12}$$

where

$$Q^2 = \omega^2 - q^2$$

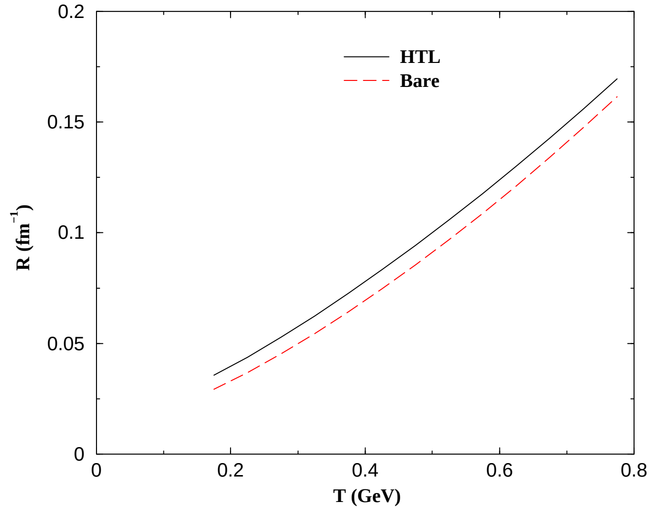

FIG. 4: (Color online) Temperature variation of the interaction rate of heavy quarks with QGP.

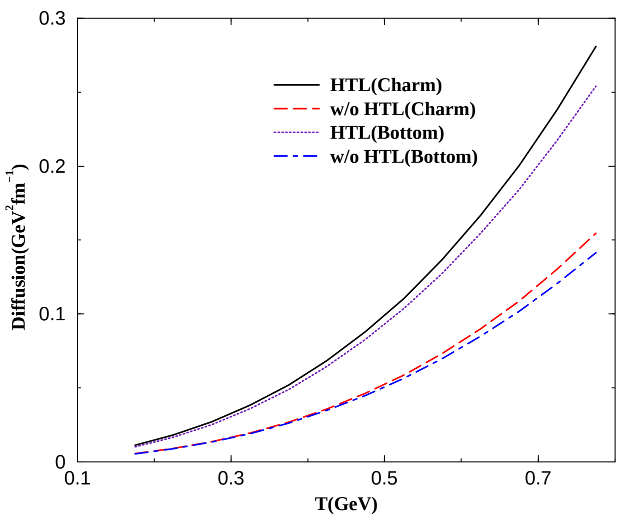

FIG. 5: (Color online) Variation of diffusion of heavy quarks of momentum 1 GeV with temperature.

$$\tilde{Q}^2 = -q^2 \tag{13}$$

Eqs. 12 and 13 are valid in the local rest frame of fluid, i.e. in a frame where $u = (1, \vec{0})$. Similarly a tensor orthogonal to u_μ can be defined,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \tag{14}$$

The full gluon propagator with momentum Q is obtained from the vacuum polarization by using Dyson-Schwinger equation:

$$\Delta^{\mu\nu} = \frac{\mathcal{P}_T^{\mu\nu}}{-Q^2 + \Pi_T} + \frac{\mathcal{P}_L^{\mu\nu}}{-Q^2 + \Pi_L} + (\alpha - 1)\frac{Q^\mu Q^\nu}{Q^2} \tag{15}$$

where α is a gauge-fixing parameter. The longitudinal tensor $\mathcal{P}_T^{\mu\nu}$ and the transverse tensor $\mathcal{P}_L^{\mu\nu}$ are defined as [26]

$$\mathcal{P}_L^{\mu\nu} = -\frac{1}{Q^2 q^2}(\omega Q^\mu - Q^2 u^\mu)(\omega Q^\nu - Q^2 u^\nu) \tag{16}$$

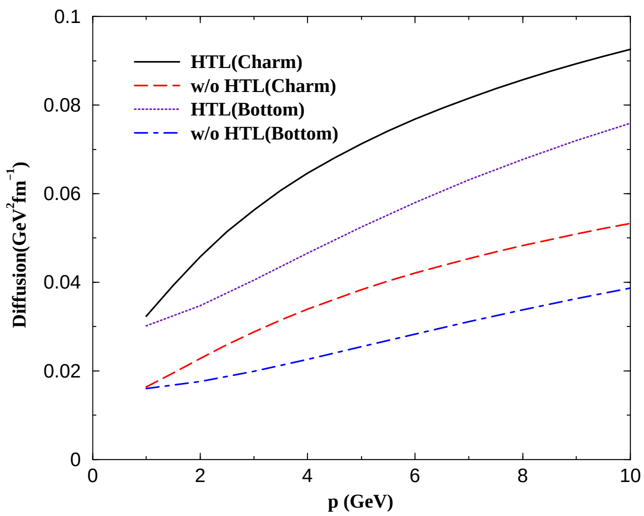

FIG. 6: (Color online) Variation of diffusion of heavy quarks with momentum in a QGP bath of temperature 300 MeV.

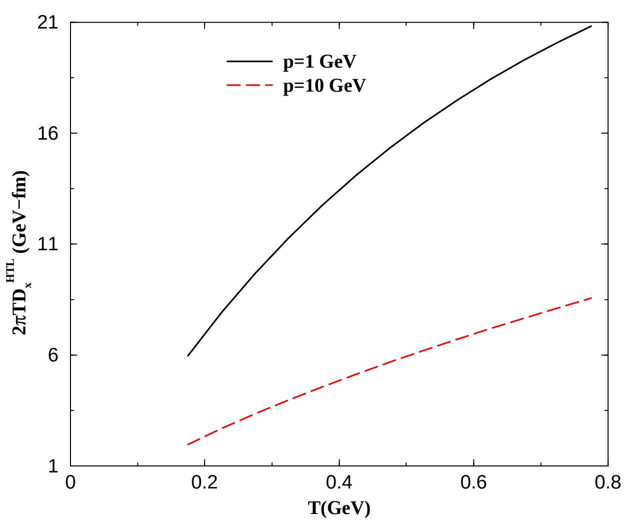

FIG. 7: Comparison of co-ordinate space diffusion of charm quark with thermal de Broglie wavelength.

$$\mathcal{P}_T^{\mu\nu} = \tilde{g}_{\mu\nu} + \frac{\tilde{Q}_\mu \tilde{Q}_\nu}{q^2} \tag{17}$$

which are orthogonal to Q^μ as well as to each other, *i.e.*

$$Q_\mu \mathcal{P}_L^{\mu\nu} = Q_\mu \mathcal{P}_T^{\mu\nu} = \mathcal{P}_{L\nu}^{\mu} \mathcal{P}_T^{\nu\rho} = 0 \tag{18}$$

But,

$$\mathcal{P}_i^{\mu\rho} \mathcal{P}_{i\nu\rho} = \mathcal{P}_{i\nu}^{\mu} \quad ,\mathrm{i = L/T} \tag{19}$$

The free gluon propagator at zero temperature is

$$D^{\mu\nu} = \left(-g^{\mu\nu} + \alpha \frac{Q^\mu Q^\nu}{Q^2}\right) \frac{1}{Q^2} \tag{20}$$

The transverse and longitudinal self-energies are given by

$$\Pi_L(Q) = (1 - x^2)\pi_L(x), \qquad \Pi_T(Q) = \pi_T(x) \tag{21}$$

where $x = \omega/q$ and scaled self-energies π_T and π_L are given by [24],

$$\pi_T(x) = m_D^2 \left[\frac{x^2}{2} + \frac{x}{4}(1-x^2)ln\left(\frac{1+x}{1-x}\right) - i\frac{\pi}{4}x(1-x^2)\right]$$
$$\pi_L(x) = m_D^2 \left[1 - \frac{x}{2}ln(\frac{1+x}{1-x}) + i\frac{\pi}{2}x\right] \tag{22}$$

Non-zero imaginary parts of the self-energies corresponds to different processes taking place in the medium.

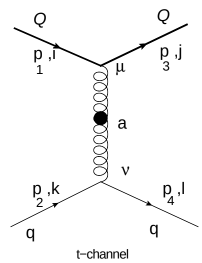

FIG. 8: $Qq \to Qq$ Feynman diagram. Bold lines are for heavy quarks(Q).

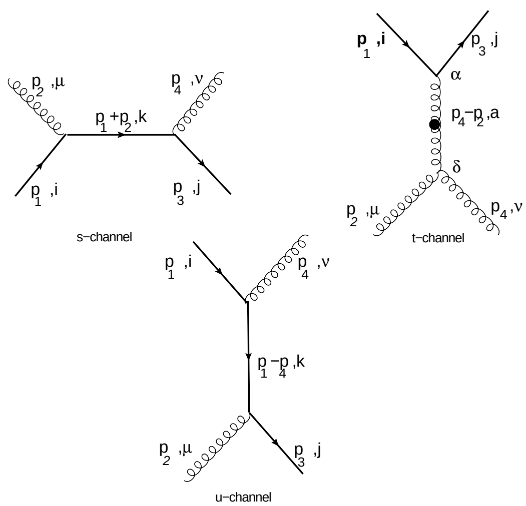

FIG. 9: $Qg \to Qg$ Feynman diagrams

A. $Qq \to Qq$ Matrix Element from HTL approximation:

The t-channel gluons that appears in the matrix element for the process $Qq \to Qq$ (Fig.8) is taken as the effective gluon propagator obtained by HTL approximation. In Figs. 8 and 9 the effective propagator will be denoted by a solid circle. We can write the amplitude in Feynman Gauge($\alpha = 1$) from fig. 8 as,

$$-iM_t = \overline{u}(p_3)(-ig\gamma^\mu t^a_{ji})u(p_1)\left[-i\Delta_{\mu\nu}\right]$$
$$\overline{u}(p_4)(-ig\gamma^\nu t^a_{lk})u(p_2) \tag{23}$$

where g is strong coupling and $g^2 = 4\pi\alpha_s$. i, j, k, l $(i \neq j,\ k \neq l)$ are quark colours and 'a' is the colour of intermediary gluon with polarizations μ, ν. After squaring and averaging over spin and colour and using Eq. 15 we get,

$$
\begin{aligned}
\frac{|M_{Qq}|^2}{4C_{Qq}g^4} &= 2\frac{p_4.\mathcal{P}_T.p_3p_2.\mathcal{P}_T.p_1}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_3p_2.\mathcal{P}_L.p_1}{(t-\Pi_L)^2} \\
&+ 2\frac{p_4.\mathcal{P}_T.p_1p_2.\mathcal{P}_T.p_3}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_1p_2.\mathcal{P}_L.p_3}{(t-\Pi_L)^2} \\
&+ 2A\frac{p_4.\mathcal{P}_L.p_3p_2.\mathcal{P}_T.p_1 + p_4.\mathcal{P}_T.p_3p_2.\mathcal{P}_L.p_1}{(t-\Pi_T)^2(t-\Pi_L)^2} \\
&+ 2A\frac{p_4.\mathcal{P}_L.p_1p_2.\mathcal{P}_T.p_3 + p_4.\mathcal{P}_T.p_1p_2.\mathcal{P}_L.p_3}{(t-\Pi_T)^2(t-\Pi_L)^2} \\
&- 2p_4.p_2\left[\frac{p_3.\mathcal{P}_T.p_1}{(t-\Pi_T)^2} + \frac{p_3.\mathcal{P}_L.p_1}{(t-\Pi_L)^2}\right] \\
&- 2p_3.p_1\left[\frac{p_4.\mathcal{P}_T.p_2}{(t-\Pi_T)^2} + \frac{p_4.\mathcal{P}_L.p_2}{(t-\Pi_L)^2}\right] \\
&+ p_3.p_1p_4.p_2\left[\frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2}\right] \\
&+ m^2\left[2\frac{p_4.\mathcal{P}_T.p_2}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_2}{(t-\Pi_L)^2}\right] \\
&- m^2\left[2\frac{p_4.p_2}{(t-\Pi_T)^2} + \frac{p_4.p_2}{(t-\Pi_L)^2}\right]
\end{aligned} \tag{24}
$$

where $C_{Qq} = \frac{2}{9}$ is the color factor, $Q^2 \equiv t = (p_1 - p_3)^2$, $A = t^2 - t(Re\Pi_T + Re\Pi_L) + Re\Pi_T\Pi_L^*$ and we have used the following relations.

$$
\begin{aligned}
\Delta^{\mu\rho}\Delta^{*\nu}_{\rho} &= \frac{\mathcal{P}_T^{\mu\nu}}{(t-\Pi_T)^2} + \frac{\mathcal{P}_L^{\mu\nu}}{(t-\Pi_L)^2} \\
|\Delta|^2 &= \Delta^{\mu\nu}\Delta^*_{\nu_\mu} = \frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2}
\end{aligned} \tag{25}
$$

Using eqs. 16, 17 we can show that

$$
p_1.\mathcal{P}_L.p_2 = p_3.\mathcal{P}_L.p_4 = p_4.\mathcal{P}_L.p_1 = p_2.\mathcal{P}_L.p_3 \tag{26}
$$

where all the calculations are done in the rest frame of fluid element.
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